Injection-locked diode laser current modulation for Pound-Drever-Hall frequency stabilization using transfer cavities

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A phase-modulated RF current source is applied to an injection-locked diode laser operating at 780 nm. This produces tunable phase-modulated sidebands of the laser suitable for stabilizing the length of an optical transfer cavity using the Pound-Drever-Hall technique. The Pound-Drever-Hall signal is antisymmetric about the lock point, despite the presence of significant diode laser amplitude modulation. The stabilized optical transfer cavity is used to frequency stabilize a 776 nm external cavity diode laser. The stability and tunability of this transfer cavity locked laser is established by observation of the hyperfine components of the ^{87}Rb $5P_{3/2}-5D_{5/2}$ transition in a vapor cell. © 2012 Optical Society of America

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Lasers are often frequency stabilized using optical cavity modes. Although stable passive cavities can be constructed [1], it is also possible to *actively* stabilize a cavity, using a second laser that is frequency stabilized using an atomic reference [2]. The cavity length can be adjusted with a high bandwidth piezoelectric actuator (PZT) [3] to keep it in resonance with the reference laser. Since the cavity *transfers* the stability of a reference laser to a target laser, it is often referred to as a "transfer cavity." The two lasers may be at very different wavelengths, provided the cavity finesse remains sufficiently high at the two wavelengths.

For typical transfer cavities, the frequency spacing of the modes is dense enough so that it is possible to place the target laser within several hundred MHz of any desired frequency. To stabilize the target laser at an arbitrary frequency, a tunable frequency shift of either the target or reference laser is necessary. This shift may be obtained by electro-optic modulators [2] or acousto-optic modulators [4].

In [5] it was demonstrated that tunable frequency shifts may also be obtained using current-modulated injection-locked diode lasers. An external cavity diode laser-frequency stabilized using an atomic reference-was used to injection lock a second diode laser. This "slave" laser was current modulated using an adjustable RF frequency. The modulation put sidebands on the laser light, and the transfer cavity was locked to one of these tunable sidebands. In this manner, by changing the RF frequency of the current modulation the transfer cavity length could be controlled, and thus the target laser frequency precisely scanned.

In a PZT-based transfer cavity, the error signals for both locking the cavity to the reference and target to the cavity can be derived by dithering the cavity length and demodulating the transmitted or reflected powers using a lock-in amplifier (as in [5]). However, PZT actuated mirrors limit the bandwidth of this technique (typically on the order of kHz). In general the feedback bandwidth must be on the order of a decade lower than the modulation frequency, and the modulation frequency is limited by the mechanical response of the PZT (although the PZT can be driven at higher frequencies the phase relationship between drive and displacement is not as well-defined). It is more desirable to modulate the incident light frequency, as in the Pound-Drever-Hall (PDH) technique [6,7], which typically allows feedback bandwidths on the order of MHz.

In this paper we demonstrate that *both* a tunable frequency shift and the modulation required for PDH locking can be obtained by applying a phase-modulated RF current to an injection-locked slave diode laser. This modulation produces tunable phase-modulated sidebands of the reference laser. By locking a transfer cavity to one of these sidebands, its length is controlled by a stable, tunable reference frequency, and the frequency of a target laser may be precisely controlled.

The mechanism for producing sidebands can be intuitively understood from the general behavior that injection-locked systems exhibit [8]: the phase difference between the injected and locked oscillator signal is $\phi = \sin^{-1}\left(\{\omega_0 - \omega_1\}/\omega_m\right)$, where ω_0 is the free running oscillator (slave) angular frequency, ω_1 is the injected signal angular frequency, and ω_m is the locking half-width (the total locking range being $2\omega_m$). As the phase difference between the injected and locked oscillator signal depends on the detuning of the injected frequency from the free running slave frequency, modulation of the slave resonance frequency will phase modulate its output.

The lasing frequency of a conventional Fabry–Perot diode laser can be rapidly changed with current. Kobayashi and

Kimura [9] demonstrated that by sinusoidally modulating the current applied to an injection-locked diode laser, they could observe frequency sidebands corresponding to phase modulation. In particular, provided the frequency content of the injection current $I(t) = I_{\mathrm{dc}} + \Delta I(t)$ is within the locking half-width ω_m, the output of the slave laser is of the form:

$$\tilde{E} = \tilde{E}_0 \exp\{j(\omega_0 t + k\Delta I(t))\}, \tag{1}$$

where $\Delta I(t)$ represents the deviation of the current from the dc value corresponding to $\omega_0 = \omega_1$, and $k \approx (\partial\omega_0/\partial I)/\omega_m$. Sinusoidal modulation of $\Delta I(t)$ produces phase modulation: $\tilde{E} = \tilde{E}_0 \exp\{j(\omega_0 t + \alpha \sin[\delta t])\}$, with sidebands in frequency space at $\omega_0 \pm \delta$ with powers of $P_1 = P[J_1(\alpha)]^2$, where P is the total power. As $\delta \to \omega_m$ this ceases to be true and the output is more accurately described as being frequency-modulated rather than phase-modulated [9]. This simple picture neglects the amplitude modulation that must accompany diode laser current modulation. We will return to this important issue after presentation of the experimental results, which unexpectedly show no adverse effects due to residual amplitude modulation.

Here we consider a current source with phase modulation at an angular frequency Ω that is expected to produce a slave laser output:

$$\tilde{E} = \tilde{E}_0 \exp\{j(\omega_0 t + \alpha \sin[\delta t + \beta \sin(\Omega t)])\}. \tag{2}$$

With $\delta \gg \Omega$ this output corresponds to tunable phase-modulated sidebands centered around $\omega_0 \pm \delta$. The cavity may be locked to one of the sidebands using the standard PDH scheme where the modulation frequency is Ω. The cavity length may then be scanned by varying δ (keeping Ω constant). If a second "target laser" is locked to another mode of the same cavity, its frequency may be controlled using δ.

This technique requires a tunable phase-modulated current source. In our implementation, a voltage controlled oscillator (VCO, Minicircuits ZX95 – 850+) is used to create a phase-modulated RF current source, with a fixed center frequency of 795 MHz, and a 10 MHz $[\Omega/(2\pi)]$ phase modulation frequency. The VCO output is mixed with a synthesized RF source with a tunable frequency of 20 MHz to 775 MHz. The output of the mixer is amplified and low-pass filtered (for rejection of the sum frequency) to obtain a tunable phase-modulated current source centered around frequencies from 775 MHz to 20 MHz $[\delta/(2\pi)]$. This signal is applied through a bias-T to an injection-locked slave diode laser (Sanyo DL7140-201S) operating at 780 nm [see Fig. 1]. The master laser is frequency stabilized to a Rb transition in a cell using FM spectroscopy [10].

An RF reference cavity was used to stabilize the center frequency of the modulated VCO. The 10 MHz phase modulation of the VCO is used to lock its center frequency (795 MHz) to this cavity using the Pound technique [11] (see Fig. 8 of [12]). An integrator varies the input voltage to the VCO based on the Pound error signal. The reference cavity is a coaxial $\lambda/4$ resonator with silver-plated surfaces, an invar inner conductor, and a copper outer conductor and endcaps. Coupling to the cavity is via a wire loop attached to an SMA connector, threaded into the voltage node endcap. The loaded and unloaded quality factors of the cavity are 3500 and 4500, respectively.

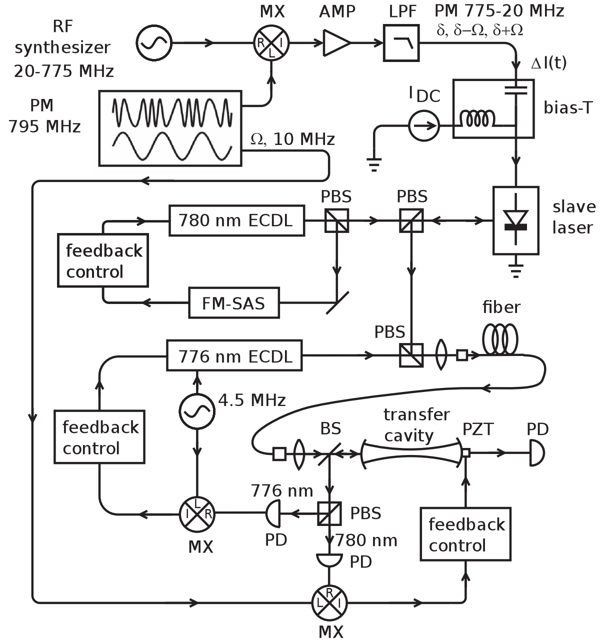

Fig. 1. PDH transfer cavity locking of a 776 nm laser using a 780 nm modulated slave laser. The 776 nm and 780 nm beams have orthogonal polarizations and are combined and separated using polarizing beam splitter cubes. The fiber is polarization preserving. AMP: amplifier, BS: nonpolarizing beam splitter, ECDL: external cavity diode laser, FM-SAS: frequency-modulated saturated absorption spectroscopy, LPF: low-pass filter, MX: mixer, PBS: polarizing beam splitter, PD: photodiode, PM: phase modulation, PZT: piezo-electric transducer.

Locking the VCO to the cavity significantly improves its long-term stability. Deviations over a 10 hour period were reduced by a factor of 10 to within 5 kHz (observed using a frequency counter with a 1 second gate). For lower drift, the cavity could be evacuated and temperature stabilized. Using an RF spectrum analyzer we observe that spurs within ≈100 kHz of the carrier are suppressed by locking the VCO and the phase noise of the locked VCO remains lower than the free VCO up to ≈2 kHz from the carrier. We expect that the high frequency phase noise performance could be improved by modifying the simple integrator feedback control.

We note that this technique (mixing with a phase-modulated, frequency stabilized VCO) is also suitable for the generation of tunable phase-modulated sidebands using electro-optic modulators, especially broadband fiber-based modulators used for transfer cavity locking (see, for example, [13]). In cases where the locked frequency stability is sufficient, it offers an inexpensive alternative to commercial synthesizers.

Before applying the phase-modulated RF we verify injection locking using a scanning Fabry–Perot cavity. By simultaneous observation of the ^{85,87}Rb, $5S_{1/2} - 5P_{3/2}$ saturated absorption spectra using both the slave and master lasers we can measure and maximize the locking bandwidth. To maintain sidebands of approximately equal strength as δ is varied, it is desirable that the locking bandwidth exceed the maximum modulation frequency (775 MHz in this case). Using 2 mW of master laser power, we obtain a locking bandwidth of $2\omega_m/(2\pi) \approx 2.8$ GHz.

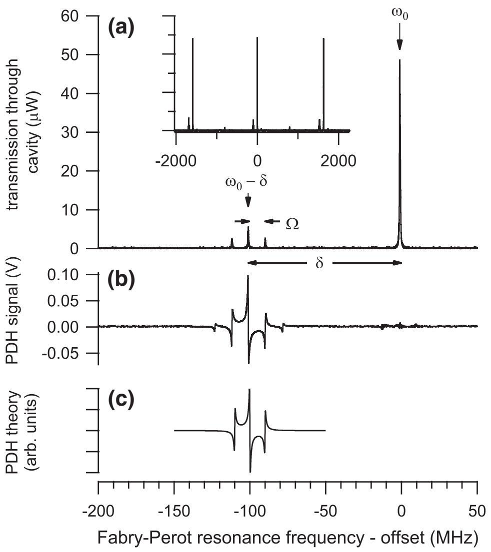

Fig. 2. (a) Fabry–Perot transmission spectra, and (b) PDH error signal for $\delta/(2\pi) = 100$ MHz observed using a scanning Fabry-Perot cavity with a free spectral range (FSR) of 1.6 GHz and finesse ($\mathscr{F}$) of 2500. No offset has been added to this signal. (c) Calculated PDH error signal shown for comparison (see, for example [12]).

With 10 mW of phase-modulated RF incident on the slave laser (measured with a directional coupler), the output of the slave laser develops sidebands at $\pm\delta$ away from the master laser frequency [see Fig. 2(a)]. These sidebands are phase modulated by Ω, allowing observation of a PDH signal [Fig. 2(b)]. The PDH signal is obtained by mixing the photodiode current obtained from reflection off the front of the cavity with the 10 MHz used to phase modulate the VCO (with an appropriate phase shift applied).

A transfer cavity could be locked to either one of the $\omega_0 \pm \delta$ sidebands. We find that they differ in amplitude and that the $\omega_0 + \delta$ sideband has an amplitude that is more susceptible to the exact setting of the dc current of the injection locked laser. A relatively long PZT travel allows a choice of cavity modes so that the more stable $\omega_0 - \delta$ sideband can always be used. We note that the difference in the behavior of the sidebands is inconsistent with our simple phase modulation picture, which neglects the amplitude modulation that must accompany current modulation.

In view of the large difference between the magnitudes of the $\omega_0 \pm \delta$ sidebands (the $\omega_0 + \delta$ sideband is barely visible in the inset to Fig. 2) it is perhaps surprising that the $\omega_0 - \delta + \Omega$ and $\omega_0 - \delta - \Omega$ sidebands have the same magnitude. It is often observed that direct current modulation of a diode laser with a *single tone* at Ω produces asymmetric sidebands, and thus imperfect PDH signals. However symmetric *secondary* sideband magnitudes are consistent with a straightforward model of the modulated slave laser output that includes amplitude modulation.

To rationalize why the secondary sideband magnitudes are equal, consider a model for the output of the slave laser that includes amplitude modulation:

$$\frac{\tilde{E}}{\tilde{E}_0} = (1 + A\sin[\phi] + B\cos[\phi])\exp\{j(\omega_0 t + \alpha\sin[\phi])\}, \quad (3)$$

where $\phi = \delta t + \beta\sin(\Omega t)$. For relatively low δ we might expect that the phase and amplitude modulation would be in phase and $B = 0$. However, this is inconsistent with our observation of asymmetric sideband amplitudes at $\omega_0 + \delta$ and $\omega_0 - \delta$ when we apply a single tone at δ (see, for example, Table 2 of [14]). There must be simultaneous amplitude and phase modulation, and a phase shift between them. This phase shift may be calculated from a detailed knowledge of the laser and the operating parameters, as Lidoyne *et al.* [15] have shown.

Using the Jacobi-Anger identity (see Section 2.22 of [16]) to expand Eq. (3), we find that the amplitude of the $\omega_0 - \delta + \Omega$ frequency component is

$$\begin{aligned}\frac{\tilde{E}_{\omega_0-\delta+\Omega}}{\tilde{E}_0} &= J_1(\alpha)J_1(\beta) - \frac{(Aj+B)}{2}J_1(\beta)J_0(\alpha)\\ &\quad + \frac{(-Aj+B)}{2}J_2(\alpha)\sum_{\ell=-\infty}^{\infty} J_\ell(\beta)J_{\ell-1}(2\beta), \quad (4)\end{aligned}$$

where J_ℓ is a Bessel function of order ℓ. The $\omega_0 - \delta - \Omega$ frequency component has the same magnitude but is of opposite sign. Thus these sidebands, which we place on the 780 nm slave laser by current modulation, are suitable for the PDH technique [6,12], which is consistent with our observations. A similar result is found for the $\omega_0 + \delta + \Omega$ and $\omega_0 + \delta - \Omega$ frequency components. We note that this result is independent of any possible phase shift between the phase and amplitude modulation.

To demonstrate the tunability of a transfer cavity locked using the modulated injection-locked slave laser, we have stabilized the frequency of a 776 nm external cavity diode laser. The transfer cavity had a FSR = 1.9 GHz and $\mathscr{F} = 350$. Its length is locked using a PM tunable sideband of the slave laser, as shown in Fig. 1. The 776 nm diode laser is then locked to a cavity resonance using PDH locking. This laser is current modulated at 4.5 MHz to provide PM sidebands suitable for locking.

The ^{87}Rb $5P_{3/2} - 5D_{5/2}$ transition [17] is used to observe the locked 776 nm laser's tunability and frequency stability. A 780 nm counterpropagating beam is sent through a Rb vapor cell exciting the $5P_{3/2}$, $F' = 3$ states, and the 776 nm absorption measured with a photodiode [Fig. 3]. We demodulate the absorption using the 4.5 MHz modulating source to produce an FM, dispersion-like absorption signal [10]. To improve signal to noise, the 780 nm beam intensity is modulated with a chopping wheel, and both the absorption and FM signals detected with a lock-in amplifier. By varying δ we may scan the frequency of the 776 nm laser over the absorption lines [see Fig. 4(a)].

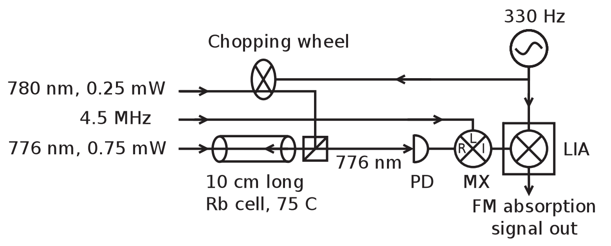

Fig. 3. Scheme to monitor 776 nm ECDL tunability and frequency stability using the ^{87}Rb, $5P_{3/2} - 5D_{5/2}$ transitions. LIA: lock-in amplifier MX: mixer, PD: photodiode.

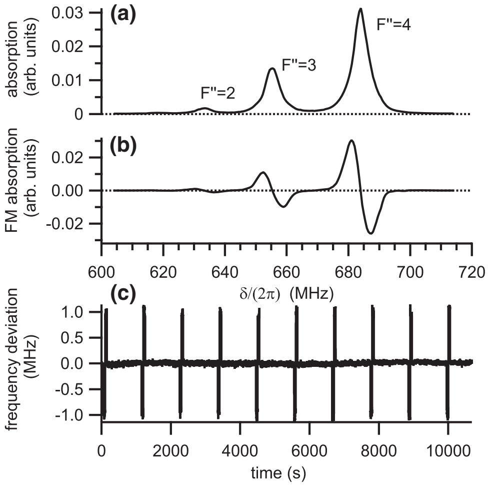

Fig. 4. (a) Absorption spectrum of the $5P_{3/2} - 5D_{5/2}$ transition of ^{87}Rb. (b) FM absorption spectrum [see Fig. 3]. (c) Frequency stability of the locked 776 nm laser as a function of time, monitored using the FM absorption signal (0.3 s time constant). The RF frequency is set to place the 776 nm laser at the $5P_{3/2}, F' = 3 \rightarrow 5D_{5/2}, F'' = 4$ transition and the FM signal is used as a frequency discriminator. Periodically stepping $\delta/(2\pi)$ up and down by 1 MHz allows conversion of the FM signal to frequency deviation.

As with Grove *et al.* [17] we can compare the observed frequency differences between absorption lines with the results of Nez *et al.* [18]. The correspondence between a frequency change in the locked laser Δf_{776} and a frequency change in the tunable sideband Δf_{780} is $\Delta f_{776}/\Delta f_{780} \approx f_{776}/f_{780} \approx 1.0055$. Using this factor, we determine from the absorption spectrum in Fig. 4(a) that the energy level difference between $5D_{5/2}$ $F'' = 4$ and $F'' = 3$ is 28.82 ± 0.03 MHz, and between $F'' = 3$ and $F'' = 2$ it is 22.7 ± 0.3 MHz. These results are consistent with 28.82 ± 0.01 MHz and 22.96 ± 0.01 MHz from [18].

As δ is varied the dc offset of the PDH signal varies, possibly due to amplitude modulation of the slave laser at 10 MHz. Although the magnitude of this offset is typically small compared to the peak PDH signal, this varying offset can introduce nonlinearities in the scanned laser frequency. In general this effect is reduced for larger δ. For example, by recording PDH spectra as a function of δ for the cavity of Fig. 2 we have established an upper bound of 50 kHz on scan nonlinearities, and a variation in the PDH signal magnitude of less than 50% for $\delta/(2\pi) = 500$ MHz to $\delta/(2\pi) = 750$ MHz. Although we have not done so, the influence of the dc offset can be reduced by deriving the PDH signal from the difference of two photodiode signals: one that measures the light incident on the cavity, and the other that measures the light reflected from the cavity. The relative powers incident on each photodiode should be adjusted to null the PDH error signal when the cavity is not resonant with the incident light.

The long-term frequency fluctuations of the 776 nm frequency stabilized laser may be monitored using the $5P_{3/2} - 5D_{5/2}$ FM signal. By setting δ to the strongest absorption line ($F'' = 4$), the FM signal [Fig. 4(b)] may be used to quantify the locked laser-frequency fluctuations [Fig. 4(c)]. Over the total time shown in Fig. 4(c) the standard deviation in the frequency is 0.026 MHz (observed using a 0.3 s time constant). This frequency stability is sufficient for many laser locking applications. For example, this system has been in regular use for several months to frequency stabilize a Ti:sapphire laser operating at 960 nm for the excitation of laser cooled atoms to Rydberg states [19]. The lock is typically maintained over several hours, usually limited by the need to relock the master laser.

In contrast to a previous transfer cavity locking setup [5] using PZT end mirror modulation to generate error signals, we find that the PDH based locks are more robust against mechanical disturbances. This improvement is due to the larger bandwidth [6,7], and effective capture range $\approx 2\Omega/(2\pi)$ of the PDH technique (the effective capture range of the PZT modulation scheme is on the order of the cavity linewidth). The ability of the transfer cavity to follow the reference laser is still ultimately limited by the PZT bandwidth (on the order of kHz). However, if the cavity has good passive mechanical isolation, the lock of the target laser to the cavity can make use of the full bandwidth of the PDH technique (on the order of MHz). This is particularly useful when frequency stabilizing external cavity diode lasers, as it eliminates the need for a prestabilization cavity [20], which might otherwise be necessary with lower-bandwidth error signals based on PZT modulation.

In summary, we have demonstrated that with the appropriate current modulation, tunable phase-modulated sidebands can be put on injection-locked diode lasers. These sidebands may be used to stabilize and vary the lengths of PZT-based optical transfer cavities using the PDH technique.

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